AUTOMATION AND MECHANIZATION OF PRODUCTION

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INCREASING THE EFFICIENCY OF VIBRATORY MECHANISMS BY EXCITING LOW-FREQUENCY RESONANCE VIBRATIONS

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The problems of increasing the operating efficiency of vibratory mechanisms by means of multiple combination parametric resonance are examined. It is proposed that this can be done with a parametric vibration exciter where the driving force is generated by the interaction of masses, which rock in the centrifugal inertial force field generated by a rotating rotor and in the field of gravity, with a working unit which elastic components secure to a base.

Vibratory feeders and conveyors [1], vibratory mechanisms which initiate the flow of loose materials, and other vibratory mechanisms are the principal components of production–flow lines conveying the components of the glass batch and cullet in sectional and machine–tank factor works.

Electrical, mechanical, and pneumatic vibratory units are used as vibrators in such mechanisms. Electromagnetic vibrators, in which ac electro- or permanent magnets are used to produce the vibrations, and electromechanical vibrators, where unbalanced masses placed on the shaft of an electric motor are used to produce vibrations, are the vibrators most widely used in the glass industry.

Most vibratory mechanisms and feeders of the glass-batch components operate in a forced-vibration regime at frequencies far from resonance, as a result of which energy is not used efficiently. This is because in such frequency ranges the vibration regime of a vibratory mechanism is insensitive to and essentially independent of changes in the parameters of the technological load (a decrease of the material level in a hopper or fluctuations of the moisture content, granulometric composition, or bulk density of the transported material).

A novel method of parametric excitation of vibrations of mechanical systems (RF Patents Nos. 2072660 and 2072661), which makes it possible to implement a multiple combination parametric resonance regime, can increase the operating efficiency of vibratory mechanisms.

The properties of a combination parametric resonance can be analyzed for the example of a dynamic model of the vibratory setup shown in Fig. 1.

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The vibratory setup (see Fig. 1*a*) contains a balanced rotor *1* with mass m_0, rigidly secured to a drive shaft *2*, whose rotation axis is oriented perpendicular to the vertical plane. The drive shaft is set in bearings which are held in the supports *3* rigidly tied to the working unit *3* with mass M_0. Elastic elements *6* secure the working unit to the base *5*. A damper *7* simulates friction from the technological load. It is assumed that the working unit undergoes only translational motion along the Oy axis. The coordinate system $Ax'y'z'$ whose origin lies at the center of the rotor and whose axes are parallel to the corresponding axes of the stationary coordinate system Oxyz moves translationally relative to the latter. In a position of static equilibrium the Az' axis coincides with the Oz axis.

The rotor has three periodically alternating open circular running tracks *8*, whose centers are shifted from the rotation axis of the rotor by the same distances $AB = l$ (see Fig. 1*b*). Rocking bodies (runners) *9* each with mass *m* are placed on the running tracks and are free to roll in their tracks.

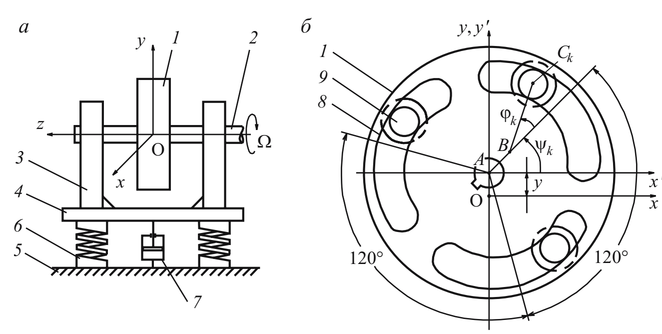

Fig. 1. Diagram of a parametric vibratory setup.

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This setup can be regarded as a mechanism for periodically changing the inertial parameters of the vibratory system in time with period $2\pi/\omega$. A periodic change in the parameters of the system is obtained by imposing on the motion of the runners nonstationary (rheonomic) constraints as a result of the uniform rotation of the rotor with angular velocity ω. The rotating rotor with the runners rocking in a centrifugal force field forms an inertial element (IE) of the parametric vibration exciter (RF Patent No. 2072661).

The dynamics of this vibratory mechanism with the rotation plane of the rotor of the IE in a horizontal position has been studied in [2 – 4]. When the rotor is in a vertical position the forces due to the weights of the runners create a nonstationary force field. The coefficients of the force function are periodic functions of time with period $2\pi/\omega$, which results in additional parametric excitation [5].

The positions of the running tracks are determined by the angles $\psi_k = \omega t + 2\pi k/N$ (N is the number of runners), and the positions of the runners are determined by the angles φ_k ($k = 1, 2, ..., N$). The rocking angles φ_k of the runners and the displacement y of the working unit are taken as the generalized coordinates of this mechanical system.

Moving along the Oy axis the working unit overcomes the elastic forces of the nonlinear springs and resistance forces, whose characteristics are given in the form

$$F_y = cy + c_1 y^3;$$
$$R_y = b\dot{y} + b_1 y^2 \dot{y},$$

where c is the total stiffness of the elastic elements, c_1 is the coefficient of nonlinearity of the elastic restoring force, b and b_1 are positive constants, and an overdot denotes differentiation with respect to the time t.

The following differential equations, governing the motion of the vibratory mechanism, with periodic coefficients were derived on the basis of Lagrange's equations of the second kind:

$$\left.\begin{aligned}&\ddot{\varphi}_k + 2(n_0 + h_0\varphi_k^2)\dot{\varphi}_k + v^2\omega^2 \sin\varphi_k = \\ &-v^2(g/l + \ddot{y})\cos(\varphi_k + \psi_k); \\ &\ddot{y} + 2(n + hy^2)\dot{y} + \omega_y^2(1 + \gamma y^2)y = \\ &\frac{m\rho_c}{M}\sum_{k=1}^{N}[(\omega + \dot{\varphi}_k)^2 \sin(\varphi_k + \psi_k) - \ddot{\varphi}_k \cos(\varphi_k + \psi_k)],\end{aligned}\right\} \quad (1)$$

where n_0 and h_0 are, respectively, the coefficients of the linear and nonlinear damping of the runners; $k = 1, 2, ..., N$, and $N = 3$; $v = (m\rho_c l/J_B)^{0.5}$ (ρ_c is the density of the glass batch; J_B — the reduced moment of inertia of a runner relative to the axis of rolling) — the relative rocking frequency of the runners; g is the acceleration of gravity; $n = b/2M$ and $h = b_1/2M$ are, respectively, the coefficients of linear and nonlinear damping of the working unit; $\omega_y = c/M$ is the partial frequency of the intrinsic vibrations of the working unit; $\gamma = c_1/c$; $\rho_c = BC_k$; $M = M_0 + m_0 + Nm$ is the total mass of the entire system.

We shall investigate the resonance properties of the system (1) for $\omega_y \approx \omega/2$.

The terms containing the second and third powers of the coordinates and their derivatives play the main role in the formation of the most important effects in the nonlinear system (1). Consequently, the trigonometric functions $\sin\varphi_k$ and $\cos\varphi_k$ in Eqs. (1) are replaced by two terms of their series expansions.

Disturbance of the position of equilibrium and self-excitation of vibrations are possible in systems whose motion is described by differential equations with periodic coefficients. The existence of such phenomena and their possible applications in electromagnetic systems are well known.

For our nonlinear system it is impossible to draw a sharp line between forced and parametric excitations. This is because a forced vibratory process in a nonlinear system can cause the corresponding parameters to change periodically. The driving force in Eqs. (1) plays the role of "oscillations of pumping." On this basis we introduce the new variable u_k defined as

$$\varphi_k = u_k + d\cos\psi_k,$$

where $d\cos\psi_k$ is a particular solution of the equation $\ddot{\varphi}_k + \nu^2\omega^2\varphi_k = -\nu^2\mu\cos\psi_k$, corresponding to forced nonresonant oscillations of the runners; $d = \mu v^2(1 - v^2)^{-1}\omega^{-2}$, and $\mu = g/l\omega_y^2$.

When the working unit is in a fixed position ($y \equiv 0$) the expression $d\cos\psi_k$ is an approximate solution of the system (1), corresponding to nonresonant oscillations of the runners. It can be shown that in this case the center of mass of the system of runners describes a circle with radius $d^2\rho_c/8$ in the direction of rotation of the rotor of the IE with respect to the coordinate system $Ax'y'z'$ [4, 5]. Its rotational angular velocity is $N\omega$ ($N = 3$). As a result of the rocking of the runners, the rotor of the IE becomes unbalanced. The rotation of the center of mass of the system of runners gives rise to a centrifugal inertial force which is transmitted to the working unit of the vibratory mechanism. The harmonic components of the force from the rocking of all runners mutually cancel, except for the harmonics whose frequencies are multiples of $N\omega$. It is these components that are transmitted to the working unit. The harmonic with frequency $N\omega$ has the largest amplitude, so than the effect of the higher-order harmonics can be neglected. Since $N\omega \gg \omega_y$, to a first approximation the effect of the harmonic with frequency $N\omega$ on the working unit outside the resonance zone can be neglected and it can be assumed that $y = 0$. Thus, the system (1) in the new variables u_k is in a position of equilibrium $u_k = 0$, $y = 0$.

The parametric resonances with can be appear in the system (1) arise near the frequencies

$$\omega = (\omega_i + \omega_j)s^{-1}, \quad i, j = 1, 2, ..., N+1, \quad s = 1, 2, ...$$

The frequencies with $i = j$, i.e., $\omega = 2\omega_i s^{-1}$, are called simple resonance frequencies and the frequencies with $i \neq j$

are called combination resonance frequencies (ω_i are the relative partial frequencies of the characteristic vibrations (RF Patent No. 2072661)).

We shall now examine the combination resonance where vibrations are excited at the frequencies ω_1 and ω_2 which are related with the excitation frequency ω by the relation

$$\omega = \omega_1 + \omega_2, \tag{2}$$

where $\omega_1 = \nu\omega$, $\omega_2 \approx \omega_y$, and $\nu = 1/2$, $\omega_y = \omega/2$.

For such tuning the partial frequencies satisfy the conditions for the excitation of combination and simple parametric resonances simultaneously.

We shall seek the solution of the system of equations (1) in the new variables, taking account of Eq. (1), in the form

$$u_k = a_k \cos(\omega_1 t + \theta_k),\ \ y = a\cos(\omega_2 t + \theta),\ \ k = 1, 2, ..., N$$

and we shall use the Bubnov – Galerkin method [6] to determine unknown amplitudes a_k and a, the phases θ_k and θ, and the generation frequencies ω_1 and ω_2.

Since the runners are identical, $a_k = a_0$, $k = 1, 2, ..., N$. By virtue of the symmetry of the IE we shall seek the solution of the transformed equations (1) for $\theta_k = 2\pi k/N$. Using the Bubnov – Galerkin method we arrive at a system of algebraic equations for a_0, a, è, ω_1, and ω_2. The solution of this system was obtained numerically by iteration using Seidel's method. The results of the numerical solution of the equations are presented in Figs. 2 and 3 with tuning $\nu = 1/2$, $\omega_y \approx \omega/2$ and the following values of the parameters of the system: $N = 3$, $\omega_y = 25\ \mathrm{sec}^{-1}$, $\tilde{n}_0 = 0.015$, $\tilde{h}_0 = \tilde{h} = 0.02$, $\varepsilon = 0.03$, $\beta_0 = 0$, $\mu = 1.57$ ($\tilde{n}_0 = n_0/\omega_y$; $\tilde{n} = n/\omega_y$; $\tilde{h}_0 = h_0/\omega_y$; $\tilde{h} = h/\omega_y$; $\varepsilon = N(m\rho_c)^2/(2J_B M)$ is a coefficient that is proportional to the ratio of the total mass of the runners to the mass of the entire system; $\beta_0 = \gamma l^2$, $\tilde{\omega} = \omega/\omega_y$, $\tilde{\omega}_1 = \omega_1/\omega_y$, $\tilde{\omega}_2 = \omega_2/\omega_y$, $\tilde{a} = a/l$.

Studying the stability of the stationary working states of the mechanism, it can be shown that the dashed curves in Fig. 2 correspond to unstable solutions.

Analysis of the amplitude – frequency characteristics showed that the resonance curves are flat and do not have the maximum characteristic of high-Q oscillatory systems at resonance of forced oscillations. Using only trigonometric nonlinearities, which are due to the characteristic features of the kinematics of the rotating IE, makes it possible to increase substantially the stability of the resonance regime of the vibrations of the working unit with linear elastic elements. Increasing the coefficient $\tilde{n}$ of linear damping of the working unit of the vibratory mechanism for small values of the linear damping $\tilde{n}_0$ of the runners enlarges the resonance zone (see Fig. 2). With a six-fold increase of the linear damping the vibrational amplitude of the working unit decreases by only a factor of 2.5. This also attests to adequate stability of the resonance regime of the vibrations. The amplitude –

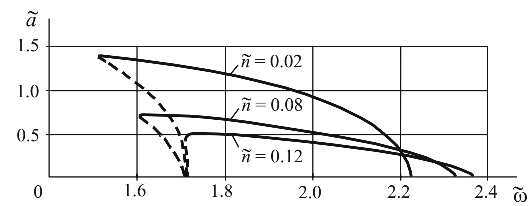

Fig. 2. Amplitude – frequency characteristics of the working unit as a function of the linear damping.

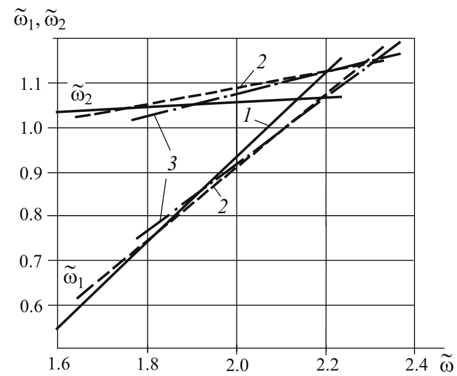

Fig. 3. Generated frequencies $\tilde{\omega}_1(\tilde{\omega})$ and $\tilde{\omega}_2(\tilde{\omega})$ versus the excitation frequencies: *1*, *2*, and *3*) $\tilde{n} = 0.02$, 0.08, and 0.12, respectively.

frequency characteristics of the runners are similar to those of the working unit.

We note that the resonance curves in Fig. 2 were constructed neglecting the stops limiting the rocking angles of the runners. It is assumed that the runners have individual running tracks which have a large rocking angle and are positioned in parallel planes.

The variation of the generated frequencies ω_1 and ω_2 as a function of the excitation frequency is shown in Fig. 3. The vibrational frequency ω_2 of the working unit remains almost constant and approximately equal to the 1 or, in dimensionless units, $\omega_2 \approx \omega/2$.

The following threshold condition must be satisfied for vibrations to exist:

$$\varepsilon > \frac{8\nu\tilde{n}\tilde{n}_0}{1-\nu}.$$

As one can see, the threshold value of the parameter ε depends strongly on the parameter ν. For example, when the rocking frequency of the runners changes from $\nu = 0.25$ to $\nu = 0.50$ the threshold value increases by a factor of 3. Consequently, for $\nu = 0.50$ there arises the problem of ensuring self-excitation of the vibrations of the vibratory machine for sufficiently small ε.

In addition, dry friction in the rotor – runner system and in the units securing the elastic elements also results in an excitation threshold. However, in the present case the high-frequency component of the vibrations converts dry friction into viscous friction [7]. Outside the resonance zone two processes occur in the vibratory system of this setup: nonreso-

nant rocking of the runners with driving frequency ω and vibrations of the working unit with frequency 3ω (co-vibrations). In the resonance zone the runners oscillate with frequency $\omega_1 \approx \nu\omega$ ($\nu = 0.5$), i.e., with a frequency approximately six times lower than the frequency of the co-vibrations of the working unit. Hence it is evident that under the conditions considered here dry friction becomes viscous friction. For this reason, the excitation threshold due to dry friction can be overcome with zero initial conditions at startup. We note that the vibratory activity of the runners promotes self-excitation of the vibratory mechanism because they are tuned to the simple (main) parametric and autoparametric resonances for any parametric excitation frequency ω. These factors guarantee that the setup will always enter a resonant vibratory regime.

When the mechanism is started up and reaches rotational frequencies in the region of instability relative to the equilibrium position $u_k \equiv 0$, $y \equiv 0$ (resonance zone), a multiple combination parametric resonance is excited. As a result of the rocking motion of the runners in the IE a definite phase relation is established between them ($\theta_k = 2\pi k/N$; $k = 1, 2, 3$; $N = 3$) and their common center of mass moves along a closed curve. It is easily shown that to a first approximation the center of mass of the system of runners with respect to the coordinate system $Ax'y'z'$ with origin at the center of the rotor in the IE will move according to the law:

$$x'_C = -\frac{\rho_c a_0}{2}\left(1 - \frac{a_0^2}{8}\right)\sin \omega_2 t;$$

$$y'_C = -\frac{\rho_c a_0}{2}\left(1 - \frac{a_0^2}{8}\right)\cos \omega_2 t,$$

where $x'_C = x_C$, $y'_C = y_C - y$; x_C and y_C are the coordinates of the center of mass of the runners.

It is evident that the center of mass in the primed coordinate system describes a circle in the direction of rotation of the rotor in the IE with frequency $\omega_2 \approx \omega/2$. The radius of the circle along which the center of mass of the runners moves depends nonlinearly on their amplitude. The circular motion of the center of mass of the runners gives rise to an unbalanced centrifugal inertial force which also rotates with frequency ω in the plane of the IE. Since the working unit of the setup is tuned to the frequency $\omega_y = \omega_2 \approx \omega/2$, the centrifugal force component with frequency ω_2 will give rise to resonance vibrations of the working unit. It can be shown that in this vibrational system the vibrations are coupled with one another. The unbalanced centrifugal inertial force arising as a result of the rocking of the runners gives rise to resonance vibrations of the working unit and the vibrations of the working unit give rise to the resonance oscillations of the runners. This bilateral interaction gives rise to a multiple combination parametric resonance. The working unit undergoes almost subharmonic vibrations of the order of 1/2, i.e., with frequency $\omega_2 \approx \omega/2$.

The principle of operation of the vibrational mechanism consists of the following. When the runners rock back and forth the rotor in the IE automatically becomes unbalanced, and therefore a centrifugal inertial force is the driving force of the parametric vibration exciter. However, in contrast to the ordinary centrifugal vibrator a parametric vibration exciter converts the rotational energy of the IE more efficiently into the mechanical vibrational energy of the working unit. The vibratory mechanism acquires new properties which are not completely obvious. The resonance zone can be expanded by increasing the damping by a definite amount, and most importantly the resonance vibrational regime for a high-Q vibratory system becomes much more stable.

The fact that the frequency of the mechanical vibrations of the working unit of such a setup can be decreased without using special frequency converters is an important advantage of such a vibratory regime. A two-fold decrease of the vibrational frequency of the working unit with the amplitude remaining unchanged decreases the transport velocity approximately by the same factor. Consequently, to maintain the transport velocity the amplitude of the vibrations must be increased by a factor of 2, which does not present a problem for a resonance setup. The overall gain in the ratio of the strength and vibration insulation of the setup consists in a two-fold decrease of the force of inertia. In addition, the stiffness of the expensive elastic system decreases four-fold.

Stromizmeritel' CJSC is developing jointly with the Nizhny Novgorod State Technical University new high-capacity energy-conserving vibratory conveyors and feeders for the glass and building materials industries on the basis of the low-frequency resonance vibratory mechanism proposed above. A resonance vibratory drive makes it easier to develop systems for automatically controlling technological processes and to attain high accuracy in dispensing and high stability in conveying loose and lumpy materials.

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